DTIC® has determined on 12/15/2010 that this Technical Document has the Distribution Statement checked below. The current distribution for this document can be found in the DTIC® Technical Report Database.

☒ **DISTRIBUTION STATEMENT A.** Approved for public release; distribution is unlimited. per AFMC AFOSR\RSL

☐ **© COPYRIGHTED**; U.S. Government or Federal Rights License. All other rights and uses except those permitted by copyright law are reserved by the copyright owner.

☐ **DISTRIBUTION STATEMENT B.** Distribution authorized to U.S. Government agencies only (fill in reason) (date of determination). Other requests for this document shall be referred to (insert controlling DoD office)

☐ **DISTRIBUTION STATEMENT C.** Distribution authorized to U.S. Government Agencies and their contractors (fill in reason) (date of determination). Other requests for this document shall be referred to (insert controlling DoD office)

☐ **DISTRIBUTION STATEMENT D.** Distribution authorized to the Department of Defense and U.S. DoD contractors only (fill in reason) (date of determination). Other requests shall be referred to (insert controlling DoD office).

☐ **DISTRIBUTION STATEMENT E.** Distribution authorized to DoD Components only (fill in reason) (date of determination). Other requests shall be referred to (insert controlling DoD office).

☐ **DISTRIBUTION STATEMENT F.** Further dissemination only as directed by (inserting controlling DoD office) (date of determination) or higher DoD authority.

Distribution Statement F is also used when a document does not contain a distribution statement and no distribution statement can be determined.

☐ **DISTRIBUTION STATEMENT X.** Distribution authorized to U.S. Government Agencies and private individuals or enterprises eligible to obtain export-controlled technical data in accordance with DoDD 5230.25; (date of determination). DoD Controlling Office is (insert controlling DoD office).

AFRL-SR-AR-TR-10-0327

REPORT DOCUMENTATION PAGE

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
16-03-2010	FINAL REPORT	FROM 01-05-2006 TO 30-11-2009

4. TITLE AND SUBTITLE

REGULAR MINIMALITY PRINCIPLE IN RELATION TO DECISION MAKING AND CATEGORIZATION

5a. CONTRACT NUMBER

5b. GRANT NUMBER

FA9550-06-1-0288

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

EHTIBAR DZHAFAROV

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

PURDUE UNIVERSITY
155 Grant Street
West Lafayette
IN 47907, USA

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR
875 N. Randolph Street
Suite 324, Room 3112
Arlington VA 22203-1768

10. SPONSOR/MONITOR'S ACRONYM(S)

AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

13. SUPPLEMENTARY NOTES

14. ABSTRACT

A comprehensive mathematical theory of dissimilarity cumulation has been developed, with its applications to both pairwise comparison judgments and categorization judgments involving stimuli of arbitrary nature and complexity. A theory of regular well-matched stimulus spaces has been developed which generalizes the principles of Regular Minimality and Regular Mediality and has deep implications for psychological and philosophical issues related to distinguishability of similar stimuli. The relation of Regular Minimality to constancy of the minima of discrimination functions and to Thurstonian-type modeling has been investigated on a high level of generality. Experiments have been conducted establishing the conformity of matching judgments/adjustments to the notion of regular well-matched stimulus spaces.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER *(Include area code)*

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18
Adobe Professional 7.0

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

DTIC Data

Purchase Request Number: FQ8671-0900823

BPN: F1ATA08340B004

Proposal Number: 06-NL-081

Research Title: REGULAR MINIMALITY PRINCIPLE IN RELATION TO DECISION MAKING AND CATEGORIZATION

Type Submission: ~~Additional Funding~~ Final Report

Inst. Control Number: FA9550-06-1-0288P00003

Institution: PURDUE UNIVERSITY

Primary Investigator: Prof Ehtibar N. Dzhafarov

Invention Ind: none

Project/Task: 2313B / X

Program Manager: Jun Zhang

Objective:

This projects aims at modeling the (arguably) most fundamental act of a cognizing system/agent, namely discriminating or telling the difference between stimuli. It will provide a noval mathematical framework for characterizing the "degree" under which two perceptual stimulus differ, based on recording the agent's performance on two-alternative forced choice tasks. This supplement will specifically ask i) whether there is a principled way of constructing the so-called dissimilarity function; and ii) how can this scaling framework be applied to categorization tasks, both within the original scope of iinvestigation.

Approach:

Traditionally, the scaling of psychological space (where perceptual stimuli are modeled as points in multi-dimensional space) has been through the algorithm of MDS - multi-dimensional scaling. The PI will adopt the Fechnerian approach, namely building up the scale through accumulating small differences (as empirically obtained through threshold judgments in a two-alternative same-difference task).

Progress:

Year: 2006 **Month:** 02

NOT required AT this TIME.

20101110217

Year: 2008 **Month:** 01

Annual accomplishments: (1) General outlines have been established for a comprehensive mathematical theory of abstract dissimilarity (Dissimilarity Cumulation) and of its main psychophysical application (Universal Fechnerian Scaling): topological and uniform properties of stimulus spaces endowed with discrimination functions have been investigated, and the distance computation scheme previously proposed for discrete spaces (such as spaces of categories) was generalized to arbitrary spaces. (2) A series of experiments was launched to establish limits of precision of the Regular Minimality principle: the idea consists in introducing a non-veridical feedback for same-di¤erent judgments which would .encourage. the observer to violate Regular Minimality in order to maximize the probability of correct responses. (3) A principal di¤erence has been established between Thurstonian-type models for same-di¤erent judgments and response process models of various kinds (a single stochastic process with absorbing boundaries, two horse-racing deterministic processes with stochastically preset criteria, etc.): the response process models, unlike the Thurstonian-type

Progress:

Year: 2008 **Month:** 01

one, are generally compatible with the conjunction of Regular Minimality with Nonconstant Self-Dissimilarity.

Year: 2008 **Month:** 06

Annual accomplishments: (1) A comprehensive mathematical theory of Dissimilarity Cumulation has been developed, with its main psychophysical application (Universal Fechnerian Scaling): the theory includes topological and uniform properties of stimulus spaces endowed with discrimination functions and a general geometry of arc-length which specializes to Finsler geometry through a series of intermediate specializations. (2) A generalization of the Regular Minimality principle has been developed which involves an arbitrary set of observation areas. Regular Minimality has been linked to .observation sorites. and shown to preclude the latter. (3) The computations of and related to Fechnerian distances have been implemented in the form of a Matlab tool, as well as a comprehensive stand-alone program which checks for Regular Minimality, computed Fechnerian distances and geodesics, and performs follow-up analyses, such as MDS and cluster analysis. (4) An experimental analysis is in progress for establishing limits of precision of the Regular Minimality principle: one series of experiments uses non-veridical feedback for same-different judgments which encourages the observer to violate Regular Minimality in order to maximize the probability of correct responses; another series is the extension of the earlier developed ping-pong adjustment paradigm to two-dimensional stimuli.

Year: 2009 **Month:** 10

(1) An extension developed of the Dissimilarity Cumulation theory to categorizationtype data, linking the theory with information geometry, and the notion of dissimilarity with that of a divergence.
(2) Experiments conducted on determining the precision limits for Matching Regularity in two-dimensional stimuli (spatial position of a dot within a circumference): all deviations recorded were below limits of plausible physiological interpretation.
(3) A theory of regular well-matched stimulus spaces developed, extending the principle of Regular Minimality and linking it to the ancient ¿sorites¿ paradox: the regularity and well-matchedness turn out to be precisely the assumptions preventing the existence of ¿paradoxical¿ soritical sequences.
(4) Several papers and one book chapter are currently under review or in press.

Year: 2010 **Month:** 03 **Final**

A comprehensive mathematical theory of dissimilarity cumulation has been developed, with its applications to both pairwise comparison judgments and categorization judgments involving stimuli of arbitrary nature and complexity. A theory of regular well-matched stimulus spaces has been developed which generalizes the principles of Regular Minimality and Regular Mediality and has deep implications for psychological and philosophical issues related to distinguishability of similar stimuli. The relation of Regular Minimality to constancy of the minima of discrimination functions and to Thurstonian-type modeling has been investigated on a high level of generality. Experiments have been conducted establishing the conformity of matching judgments/adjustments to the notion of regular well-matched stimulus spaces.

Final Performance Report on AFOSR grant FA9550-06-1-0288 (reporting period May 1, 2006 to November 30, 2009)

Ehtibar N. Dzhafarov (PI), Purdue University

Purdue University was awarded the AFOSR grant "Regular Minimality principle in relation to decision making and categorization" (Grant No.: FA9550-06-1-0288). The project was initially funded for the period of May 1, 2006 to November 30, 2008 with the total amount of $311,076; it was extended until November 30, 2009 with the supplementary funding in the amount of $90,494. This report summarizes the progress made throughout the project period.

1 Progress Report

1.1 General Theory of Dissimilarity Cumulation

A new mathematical theory, of dissimilarity cumulation, has been developed; based on this theory, a radical extension of Fechnerian Scaling has been proposed, a theory dealing with the computation of subjective distances from pairwise discrimination probabilities. The new theory, called Universal Fechnerian Scaling, is applicable to all conceivable stimulus spaces (including discrete and continuous ones) subject to the following two assumptions: (A) the discrimination probabilities satisfy the Regular Minimality law; and (B) the canonical psychometric increments of the first and second kind are dissimilarity functions. Given a stimulus space $\mathfrak{S}$, a function $D : \mathfrak{S} \times \mathfrak{S} \to \mathbb{R}$ is a dissimilarity function if it has the following properties:

$\mathcal{D}1$ (*positivity*) $D\mathbf{ab} > 0$ for any distinct $\mathbf{a}, \mathbf{b} \in \mathfrak{S}$;

$\mathcal{D}2$ (*zero property*) $D\mathbf{aa} = 0$ for any $\mathbf{a} \in \mathfrak{S}$;

$\mathcal{D}3$ (*intrinsic uniform continuity*) for any $\varepsilon > 0$ one can find a $\delta > 0$ such that, for any $\mathbf{a}, \mathbf{b}, \mathbf{a}', \mathbf{b}' \in \mathfrak{S}$,

$$\text{if } D\mathbf{aa}' < \delta \text{ and } D\mathbf{bb}' < \delta, \text{ then } |D\mathbf{a}'\mathbf{b}' - D\mathbf{ab}| < \varepsilon;$$

$\mathcal{D}4$ (*chain property*) for any $\varepsilon > 0$ one can find a $\delta > 0$ such that for any chain $\mathbf{aXb}$,

$$\text{if } D\mathbf{aXb} < \delta, \text{ then } D\mathbf{ab} < \varepsilon.$$

If $\mathbf{X} = \mathbf{x}_1 \ldots \mathbf{x}_k$, the expression $D\mathbf{aXb}$ refers to the sum of the dissimilarities between the successive elements of the chain $\mathbf{aXb} = \mathbf{ax}_1 \ldots \mathbf{x}_k\mathbf{b}$ (cumulated dissimilarity, $D\mathbf{ax}_1 + \ldots + D\mathbf{x}_k\mathbf{b}$). The subjective (Fechnerian) distance $G\mathbf{ab}$ from $\mathbf{a}$ to $\mathbf{b}$ is defined as the infimum of $D\mathbf{aXb}$ across all possible chains $\mathbf{X}$ (including the empty chain) inserted between $\mathbf{a}$ and $\mathbf{b}$ (the overall, symmetrical metric between $\mathbf{a}$ to $\mathbf{b}$ is defined as $G\mathbf{ab} + G\mathbf{ba}$).

The four properties $\mathcal{D}1$-$\mathcal{D}4$ have been shown to be mutually independent. Any conventional, symmetric metric is a dissimilarity function. A quasimetric (satisfying all metric axioms except for symmetry) is a dissimilarity function if and only if it is symmetric in the small (i.e., if for any $\varepsilon > 0$ one can find a $\delta > 0$ such that $M\mathbf{ab} < \delta$ implies $M\mathbf{ba} < \varepsilon$). It has been proposed to reserve the term metric (not necessarily symmetric) for such quasimetrics. The distance $G\mathbf{ab}$ induced by $D\mathbf{ab}$ is a metric. A real-valued binary function satisfies the chain property if and only if whenever its value is sufficiently small it majorates some quasimetric and converges to zero whenever this quasimetric does. The function is a dissimilarity function if, in addition, this quasimetric is a metric with respect to which the function is uniformly continuous. The topology and uniformity induced by $G\mathbf{ab}$ and $D\mathbf{ab}$ coincide. A stimulus space endowed with D (hence also with G) is topologically characterized as a completely normal space. One of the most basic properties of G is as follows: for any $\mathbf{a}, \mathbf{b} \in \mathfrak{S}$ and any sequence $\mathbf{X}_n$ of chains, if $D\mathbf{aX}_n\mathbf{b} \to G\mathbf{ab}$

then $Ga\mathbf{X}_n\mathbf{b} \to G\mathbf{ab}$. If the stimulus space $\mathfrak{S}$ is finite, the computation of G from D can be viewed as a data-analytic procedure of "correcting" dissimilarities into distances (as an alternative to nonmetric Multidimensional Scaling): the procedure simply replaces the dissimilarity between any two stimuli with the shortest length across all chains of stimuli connecting them. It has been shown that this procedure can be equivalently described as a series of recursive corrections for violations of the triangle inequality across all triples of stimuli considered in an arbitrary order.

In arc-connected spaces the notion of the cumulated dissimilarity $D\mathbf{aXb}$ has been used to define the notion of a path length, $D\mathbf{f}$, as the limit inferior of the lengths of chains converging to the path $\mathbf{f}$ in some well-defined sense. The class of converging chains is broader than that of converging inscribed chains. Most of the fundamental results of the metric-based path length theory (additivity, lower semicontinuity, etc.) turn out to hold in the general dissimilarity-based path length theory. This shows that the triangle inequality and symmetry are not essential for these results, provided one goes beyond the traditional scheme of approximating paths by inscribed chains. The most fundamental property of the path length is that $D\mathbf{f} = G\mathbf{f}$ (note that G too is a dissimilarity, so the definition of $G\mathbf{f}$ involves no new concepts). An important role in the theory is given to the notion of a space with intermediate points: it generalizes (and specializes to when the dissimilarity is a metric) the notion of a convex space in the sense of Manger. A space is with intermediate points if for any distinct $\mathbf{a}$, $\mathbf{b}$ there is a different from them point $\mathbf{m}$ such that $D\mathbf{am} + D\mathbf{mb} \leq D\mathbf{ab}$. In such spaces the metric G induced by the dissimilarity D is intrinsic: $G\mathbf{ab}$ coincides with the infimum of lengths of all arcs connecting $\mathbf{a}$ to $\mathbf{b}$.

The general, dissimilarity-based theory of path length has been used to construct the notion of a smooth path, defined by the property that the ratio of the dissimilarity between its points to the length of the subtended fragment of the path tends to unity as the points gets closer to each other. We consider a class of stimulus spaces in which for every path there is a series of piecewise smooth paths converging to it pointwise and in length; and a subclass of such spaces where any two sufficiently close points can be connected by a smooth geodesic in the small. These notions are used to construct a broadly understood Finslerian geometry of stimulus spaces representable by regions of Euclidean n-spaces. With an additional assumption of comeasurability in the small between the canonical psychometric increments of the first and second kind, this establishes a link between Universal Fechnerian Scaling and Multidimensional Fechnerian Scaling in Euclidean n-spaces. The latter was a starting point (in 1999) for the PI's theoretical program generalizing Fechner's idea that sensation magnitudes can be computed by integrating a local discriminability measure.

1.2 Theoretical Analysis and Generalizations of the Notion of Regular Minimality

The notion of Regular Minimality for same-different judgments has been generalized to encompass all possible forms of matching, $x_1^{(\omega_1)}\mathrm{M}x_2^{(\omega_2)}$(read as 'stimulus with value x_1 in stimulus area ω_1 is matched by stimulus with value x_2 in stimulus area ω_2'). Thus, a stimulus with a given shape (x_1) presented on the left or chronologically first (ω_1) can be matched by a stimulus with a shape x_2 presented on the right or chronologically second (ω_2). For same-different comparisons, $x_1^{(\omega_1)}\mathrm{M}x_2^{(\omega_2)}$ is defined to mean

$$\psi\left(x_1^{(\omega_1)}, x_2^{(\omega_2)}\right) = \min_z \psi\left(x_1^{(\omega_1)}, z^{(\omega_2)}\right),$$

where

$$\psi(x^{(\alpha)}, y^{(\beta)}) = \Pr[x^{(\alpha)} \text{ and } y^{(\beta)} \text{ are judged to be different}].$$

For greater-less comparisons, $x_1^{(\omega_1)}\mathrm{M}x_2^{(\omega_2)}$ means

$$\gamma\left(x_1^{(\omega_1)}, x_2^{(\omega_2)}\right) = \frac{1}{2},$$

where

$$\gamma(x^{(\alpha)}, y^{(\beta)}) = \Pr[y^{(\beta)} \text{ is judged to be greater than } x^{(\alpha)}].$$

For matching adjustments, $x_2^{(\omega_2)}$ in $x_1^{(\omega_1)}\mathrm{M}x_2^{(\omega_2)}$ is a measure of central tendency of all $z^{(\omega_2)}$ judged to match $x_1^{(\omega_1)}$. Stimulus areas form an arbitrary set Ω (e.g., various locations for shapes), whereas the stimulus values in each stimulus area form an arbitrary set S (a set of shapes). A stimulus space $(S \times \Omega, \mathrm{M})$ is called a well-matched space if, for any sequence $\alpha, \beta, \gamma \in \Omega$ and any $a^{(\alpha)} \in S \times \Omega$, there is a well-matched sequence $a^{(\alpha)}, b^{(\beta)}, c^{(\gamma)}$ (i.e., a sequence in which any two elements match). $(S \times \Omega, \mathrm{M})$ is a regular space if, for any $a^{(\omega)}, b^{(\omega)}, c^{(\omega')} \in S \times \Omega$ with $\omega \neq \omega'$,

$$a^{(\omega)}\mathrm{M}c^{(\omega')} \wedge b^{(\omega)}\mathrm{M}c^{(\omega')} \Longrightarrow a^{(\omega)}\mathrm{E}b^{(\omega)},$$

where $a^{(\omega)}\mathrm{E}b^{(\omega)}$ is the equivalence relation defined as follows: $a^{(\omega)}\mathrm{E}b^{(\omega)}$ if for any $\mathrm{c}^{(\iota)} \in S \times \Omega$, we have

$$\mathrm{c}^{(\iota)}\mathrm{M}a^{(\omega)} \iff \mathrm{c}^{(\iota)}\mathrm{M}b^{(\omega)}.$$

The notion of a regular well-matched space is the generalization of the notions of Regular Minimality and Regular Mediality previously introduced by the PI for the case of just two stimulus areas (or "observation areas") and under the assumption that $a^{(\omega)}\mathrm{E}b^{(\omega)}$ means $a = b$.

It has been shown that the notion of regular well-matched spaces is central for dissolving the ancient philosophical paradox called "sorites" (ascribed to Eubulides, 4th century BCE). According to the pairwise-comparison version of sorites, a stimulus set may contain sequences of stimuli in which any two successive stimuli are not discriminable while the first and the last one are. This hypothesis, often described as an empirical fact, motivates such well-known theoretical constructs as semiorders and interval orders. A rigorous analysis of the notion of (in)discriminability, however, shows that the hypothesis has no empirical justification. Moreover, it is ruled out in regular well-matched spaces. A "logical" or plausible justification for the comparative sorites is usually found in the classificatory form of sorites, according to which if a stimulus causes a response, then a sufficiently close stimulus must cause precisely the same response. This argument has been shown to be untenable on arguably the highest possible level of abstraction in defining the notion of "sufficiently close stimuli," the level of Fréchet's proto-topological V-spaces.

New results have been obtained for the relationship between Regular Minimality and Constant Self-Dissimilarity, i.e., the property that $\psi\left(x_1^{(\omega_1)}, x_2^{(\omega_2)}\right)$ is constant across all matching pairs $\left(x_1^{(\omega_1)}, x_2^{(\omega_2)}\right)$. Thus, if each $x_1^{(\omega_1)}$ has a single matching stimulus in ω_2 (called the Point of Subjective Equality, PSE, for $x_1^{(\omega_1)}$), and if each $x_2^{(\omega_2)}$ has a single matching stimulus in ω_1 (called the PSE for $x_2^{(\omega_2)}$), then ψ complies with Regular Minimality if both its minimum level functions (restrictions of ψ to the PSE functions) are constant, or if one of them is constant and the corresponding PSE function is onto. Also, for a continuous function with continuous PSE functions (on Hausdorff, first countable, connected stimulus spaces), if the range of a PSE function is open, then its constancy implies Regular Minimality. PI's earlier results on the well-behavedness conditions under which Regular Minimality implies Constant Self-Dissimilarity have been generalized to Hausdorff arc-connected spaces. The generalization employs the notion of the smallest transitively and topologically closed extension of a relation, and this notion is analyzed by means of a transfinite-recursive construction. This may be the first application of transfinite numbers in mathematical behavioral sciences.

PI's earlier results relating to each other Regular Minimality, Constant Self-Dissimilarity, and Thurstonian-type models have also been greatly extended and improved. A Thurstonian-type model for pairwise comparisons is any model in which the response (e.g., "they are the same" or "they are different") to two stimuli being compared depends, deterministically or probabilistically, on the realizations of two randomly varying representations (perceptual images) of these stimuli. The two perceptual images in such a model may be stochastically interdependent but each has to be selectively dependent on its stimulus. It has been shown that every discrimination probability function can be generated by an appropriately chosen Thurstonian-type model with deterministic decision rule and independent perceptual images; and that every Thurstonian-type model with probabilistic decisions is equivalent, in the sense of generating the same function ψ , to a model with deterministic decisions. These statements hold for stimulus spaces of arbitrary nature. Assuming that stimuli form Hausdorff arc-connected topological spaces, by imposing certain restrictions ("well-behavedness constraints") on the components of Thurstonian-type models it has been established that a Thurstonian-type model subject to these constraints cannot generate a continuous discrimination probability functions which simultaneously (a) satisfies Regular Minimality with a homeomorphic PSE function and (b) has nonconstant minima.

1.3 Experimental Analysis of Regular Minimality

Experimental work has been conducted to gauge the limits of precision with which empirical matches comply to the hypothesis of regular well-matchedness (for two fixed stimulus areas). The previous experimental work by the PI involved unidimensional adjustments (lengths of two horizontal or one horizontal and one vertical light segments) in the "ping-pong" paradigm. This paradigm, also used in the present experiments, can be described by the following algorithm: in trial 1, $y^{(2)}$-stimulus (in the second stimulus area) is adjusted until it matches a fixed value of $x^{(1)}$-stimulus (in the first stimulus area); in trials 2, 4, 6, ..., $y^{(2)}$ remains fixed at the value achieved in the previous trial, but at the beginning of the trial the experimenter changes the value of $x^{(1)}$ in a random way so that it no longer matches $y^{(2)}$, and the participant adjusts $x^{(1)}$ until the match is restored; in trials 3, 5, 7, ..., the situation

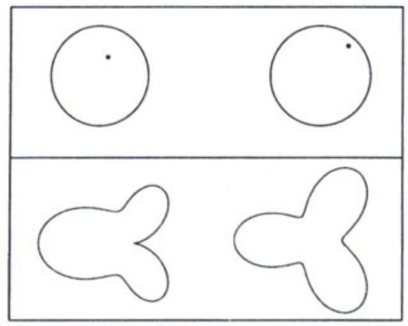

Figure 1: Examples of stimuli used in experiments 1 (top) and 2 (bottom). In Experiment 1 the positions of the two dots were adjusted to alternately match each other. In Experiment 1 the positions of the two dots the shapes of the two "floral" shapes were adjusted to alternately match each other. In either case in each trial a participant controlled two parameters of the right or of the left stimulus (x,y coordinates in Experiment 1 and the amplitudes K_a, K_b in Experiment 2) by rotating a trackball.

reverses: throughout the trial $x^{(1)}$ remains fixed at the value achieved in the previous trial, but at the beginning of the trial the experimenter changes the value of $y^{(2)}$ in a random way so that it no longer matches $x^{(1)}$, and the participant adjusts $y^{(2)}$ until the match is restored; this procedure is repeated many times (a few thousand times in the experiments conducted) forming a series, and the experiment consists of several such series starting at the same $x^{(1)}$.

The new experiments have been conducted with stimuli whose values are two-dimensional (see Figure 1). In one experiment this two-dimensional value was the location of a dot within a circle (with two dots within two circles, on the left and on the right, being adjusted to each other alternately). In the other experiment the stimuli were two, left and right, flower-like shapes created by means of the formula

$$r(\theta) = R + K_a \cos a\theta + K_b \cos b\theta,$$

where (r, θ) are polar coordinates (with the respect to the centers of the two shapes), R is the mean radius, a and b integer frequencies (1 and 3 cycles per 2π in the completed experiments), and $|K_a| + |K_b| \leq R$ (the coefficients and R are measured in pixels, or sec of arc). The results (see Figures 2 and 3, for one participant) are shown as the distributions of first-order differences between adjustments, i.e., differences between successive adjustments of $y^{(2)}$ in trials 1, 3, 5, etc., and between successive adjustments of $x^{(1)}$ in trials 2, 4, 6, etc., each difference being shown separately for the two coordinates: the horizontal and vertical positions of the dots in the first experiment, and the coefficients K_a and K_b in the second experiment. The regular well-matchedness in this analysis should manifest itself in symmetrical distributions with mean and median equal to zero, and this hypothesis is statistically supported by the data: the means and medians of the first-order differences do not exceed a few sec of arc (well below physiologically meaningful limits), and the deviations from the symmetry in the distributions of the first-order differences are well within the limits of statistical noise.

1.4 Categorization and Dissimilarity Cumulation

The application of the dissimilarity cumulation theory to categorization links this theory to information geometry. In the categorization paradigm each stimulus is represented by a categorical distribution $\mathbf{x} = (x_1, \ldots x_k)$, where x_i denotes the probability of classifying a stimulus into the ith category among an exhaustive list of $k \geq 1$ mutually exclusive categories. In information geometry the space of categorical distributions is structured by endowing it with divergences, an important class whereof is formed by the generalized Kullback-Leibler divergences,

$${}_{\alpha}(\mathbf{x}||\mathbf{y})_{\beta} = \frac{1-\alpha}{(1-\beta)^2}(\mathbf{x}||\beta\mathbf{x} + (1-\beta)\,\mathbf{y}) + \frac{\alpha}{(1-\beta)^2}(\mathbf{y}||\beta\mathbf{y} + (1-\beta)\,\mathbf{x}),$$

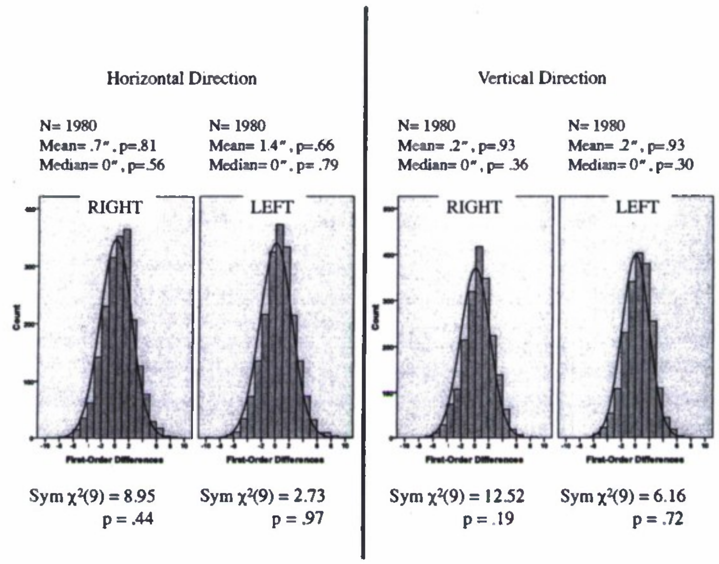

Figure 2: Experiment 1, "ping-pong" matching of the positions of two dots within two circles by one participant. The p-value for the chi-square test pertains to the null-hypothesis that the distribution is symmetric around zero, the p-values at the mean and median are for the null-hypotheses that the corresponding population parameters are not different from zero. The bin width is 1 pixel $\approx 60''$ arc.

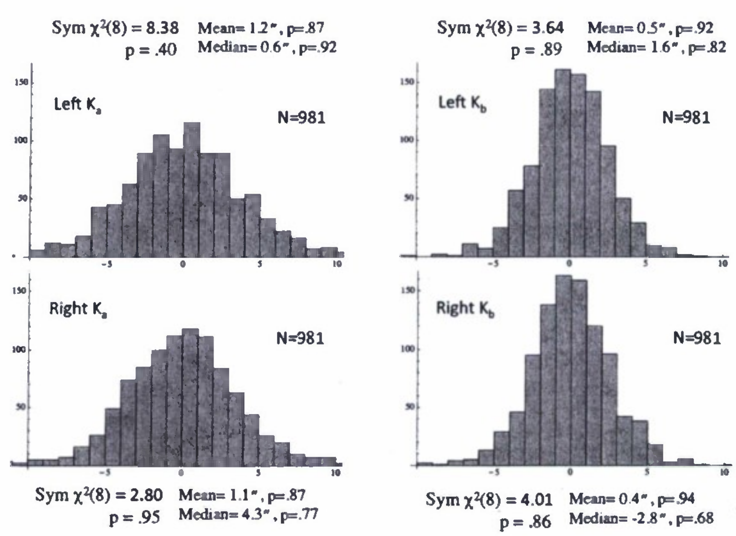

Figure 3: Experiment 2, "ping-pong" matching of two floral shapes by one participant. The format and the meaning of the p-values are the same as in Figure 2.

where $\alpha \in \left[0, \frac{1}{2}\right], \beta \in \left[0, \frac{1}{2}\right]$, and

$$(\mathbf{a}||\mathbf{b}) = \frac{1}{2}\sum_{i=1}^{k} a_i \log \frac{a_i}{b_i}.$$

To exclude infinite values it has been assumed that the class $\mathfrak{S}$ of categorical distributions $\mathbf{x} = (x_1, \ldots x_k)$ for a given class of stimuli (e.g., aerial photographs of terrains classified by pilots) is a subset of

$$\mathfrak{C}_c^{(k)} = \left\{(x_1, \ldots x_k) : \sum_{i=1}^{k} x_i = 1, x_1 > c, \ldots, x_k > c\right\},$$

where $c \in \left[0, \frac{1}{k}\right[$ (in information geometry $\mathfrak{S}$ is usually identified with $\mathfrak{C}_{c=0}^{(k)}$). It has been shown that $D_{\alpha,\beta}\mathbf{xy} = \sqrt{\alpha\,(\mathbf{x}||\mathbf{y})_\beta}$ is a dissimilarity function for any $c > 0$, but the theory of dissimilarity cumulation also applies to $c = 0$ if the intersections $\mathfrak{C}_c^{(k)} \cap \mathfrak{S}$ are metric subspaces of $\mathfrak{S}$ for some sequence $c_1, c_2, \ldots$ converging to zero (being a metric subspace here means that for any two points $\mathbf{a}, \mathbf{b} \in \mathfrak{C}_{c_i}^{(k)} \cap \mathfrak{S}$, $G_{\alpha,\beta}\mathbf{ab}$ is the infimum of $D_{\alpha,\beta}\mathbf{aXb}$ over all chains $\mathbf{X}$ whose elements are confined to $\mathfrak{C}_{c_i}^{(k)}$). In the latter case (with $c = 0$) $D_{\alpha,\beta}\mathbf{xy}$ is called a weak dissimilarity function. Of the theoretically interesting values of α (0 and $\frac{1}{2}$) and β (0 and $\frac{1}{2}$), only the square root of the symmetric Kullback-Leibler divergence ($\alpha = \frac{1}{2}$, $\beta = 0$) forms a dissimilarity (or weak dissimilarity) function that renders $\mathfrak{S}$ a space with intermediate points. In particular, if $\mathfrak{S}$ is spherically convex in the coordinates $\left(\sqrt{x_1}, \ldots \sqrt{x_k}\right)$, the value of $G\mathbf{ab}$ coincides with the length $C\mathbf{ab}$ of the great circle arc connecting $\mathbf{a}$ and $\mathbf{b}$. The dissimilarity cumulation theory therefore generalizes (and "corrects") the standard information-geometric approach, in which the great circle arc solution is used indiscriminately for all α, β, even if $D_{\alpha,\beta}\mathbf{ab}$ is shorter than $C\mathbf{ab}$ (as it is, e.g., in the case of the symmetrized Jensen-Shannon divergence, $\alpha = \frac{1}{2}$, $\beta = \frac{1}{2}$).

1.5 Related and Future Work

Some of the issues discussed in Section 1.2 (Thurstonian-type modeling) lead to the problem of selective probabilistic causality ("selective influence") under stochastic interdependence. The new results obtained by the PI in this line of research are not reported here as they more directly fall within the scope of another AFOSR-funded project (FA9550-09-1-0252) in which the PI currently participates.

The issues discussed in Section 1.4 turn out to be mathematically highly non-trivial: there remain many open theoretical questions, such as the degree of restrictiveness of the notion of weak dissimilarity function and the various ways of dealing with conjoint independent classifications. The topic of categorization in the context of dissimilarity cumulation and information geometry deserves to be pursued in further projects. The class of issues involved has considerable practical importance, both in the sense of practical applications and in the sense of the greater practicality of categorization experiments over pairwise comparison ones.

2 Other Information

2.1 Development of Human Resources

At various stages the grant was used to support two graduate students, Wasin Rujikietgumjorn and Lacey Perry. The support has enabled them to work towards their doctoral dissertations. Wasin Rujikietgumjorn has been instrumental in developing the computer program FSCAMDS and he wrote software and maintained hardware for experimental work. Lacey Perry conducted and statistically analyzed experiments. The grant was also used by the PI to supervise an honors program undergraduate student, Leigha Doherty. Materials related to this grant have been used by the PI in teaching of several graduate courses at Purdue.

2.2 Collaborations

The grant was used to support collaborative work of the PI with Janne Kujala of University of Jyväskylä, Finland, and Ali Ünlü of University of Dortmund, Germany. These collaborations resulted in several publications, conference presentations, and several ongoing projects.

2.3 Distinctions

In Fall semester of 2009 the PI was appointed a fellow of the Swedish Collegium for Adavanced Studies in Uppsala, Sweden. In 2008 the PI was an invited speaker at the International Conference on Riemann-Finsler Geometry and at the European Mathematical Psychology Group Meeting.

3 Publications and Presentations

3.1 Publications Attributed to the Grant

Dzhafarov, E.N., & Colonius, H. (2007). Dissimilarity Cumulation theory and subjective metrics. Journal of Mathematical Psychology, 51, 290–304.

Dzhafarov, E.N. (2008). Dissimilarity cumulation theory in arc-connected spaces. Journal of Mathematical Psychology, 52, 73–92.

Dzhafarov, E.N. (2008). Dissimilarity cumulation theory in smoothly-connected spaces. Journal of Mathematical Psychology, 52, 93–115.

Kujala, J.V., & Dzhafarov, E.N. (2008). On minima of discrimination functions. Journal of Mathematical Psychology, 52, 116–127.

Kujala, J.V., & Dzhafarov, E.N. (2008). Testing for selectivity in the dependence of random variables on external factors. Journal of Mathematical Psychology, 52, 128–144.

Dzhafarov, E.N. (2008). An ancient paradox for discrimination judgments. In B.A. Schneider & B.M. Ben-David (Eds) Fechner Day 2008 (pp. 41-46). Mt. Toronto, CA: Minuteman Press.

Dzhafarov, E.N. (2009). Corrigendum to: "Dissimilarity cumulation theory in arc-connected spaces" [Journal of Mathematical Psychology 47 (2003) 205–219 52 (2008) 7392]

Ünlü, A., Kiefer, T., & Dzhafarov, E.N. (2009). Fechnerian Scaling in R: The package fechner. Journal of Statistical Software, 31, 1-24. (URL for the paper and software: http://www.jstatsoft.org/v31/i06/.)

Kujala, J.V., & Dzhafarov, E.N. (2009). A new definition of well-behaved discrimination functions. Journal of Mathematical Psychology, 53, 593–599.

Kujala, J.V., & Dzhafarov, E.N. (2009). Regular Minimality and Thurstonian-type modeling. Journal of Mathematical Psychology, 53, 486–501.

Dzhafarov, E.N. (2009). Review of "Sensory Neuroscience: Four Laws of Psychophysics" by Josef J. Zwislocki, Springer, (2009). ix+170 pp., Index. Journal of Mathematical Psychology, 53, 600–602.

Dzhafarov, E.N., & Dzhafarov, D.D. (2010). Sorites without vagueness I: Classificatory sorites. Theoria, 76, 4-24.

Dzhafarov, E.N., & Dzhafarov, D.D. (2010). Sorites without vagueness II: Comparative sorites. Theoria, 25-53.

Dzhafarov, E.N. (in press as of 2009). Mathematical foundations of Universal Fechnerian Scaling. In In B. Berglund, G.B. Rossi, J. Townsend, & L. Pendrill (Eds). Measurements With Persons. New York: Taylor and Francis.

Dzhafarov, E.N., & Dzhafarov, D.D. (in press as of 2009). The sorites paradox: A behavioral approach. In J. Valsiner and L. Rudolph (Eds). Mathematical Models for Research on Cultural Dynamics: Qualitative Mathematics for the Social Sciences. Routledge: London.

Dzhafarov, E.N. (in press as of 2009). Dissimilarity Cumulation as a procedure correcting for violations of triangle inequality. Journal of Mathematical Psychology.

Dzhafarov, E.N. (in press as of 2010). Dissimilarity, quasidistance, distance. Journal of Mathematical Psychology.

Dzhafarov, E.N., & Kujala, J.V. (under review as of 2010). The Joint Distribution Criterion and the Distance Tests for selective probabilistic causality

3.2 Technical Reports Attributed to the Grant

Kujala, J.V., & Dzhafarov, E.N. (2007). Distance tests and Cosphericity tests for selective influence. Technical Reports of the Purdue Mathematical Psychology Program, #07-1.

Kujala, J.V., & Dzhafarov, E.N. (2007). Regular Minimality and constancy of the minima of discrimination functions. Technical Reports of the Purdue Mathematical Psychology Program, #07-2.

3.3 Conference Presentations Attributed to the Grant

Dzhafarov, E.N., & Colonius, H. (2006, July). From discrimination to distance through dissimilarity. Meeting of the Society for Mathematical Psychology (Vancouver, Canada).

Kujala, J.V., & Dzhafarov, E.N. (2006, July). Regular Minimality principle and well-behaved Thurstonian-type models. Meeting of the Society for Mathematical Psychology (Vancouver, Canada).

Colonius, H., Diederich, A., & Dzhafarov, E.N. (2006, July). Audio-visual integration of letters and speech: A Fechnerian Scaling analysiss. Meeting of the Society for Mathematical Psychology (Vancouver, Canada).

Dzhafarov, E.N., & Colonius, H. (2006, September). Universal Fechnerian Scaling. European Mathematical Psychology Group Meeting (Brest, France).

Kujala, J.V., & Dzhafarov, E.N. (2006, September). Regular Minimality principle. European Mathematical Psychology Group Meeting (Brest, France).

Colonius, H., & Dzhafarov, E.N. (2006, September). Audio-visual integration of letters and speech. European Mathematical Psychology Group Meeting (Brest, France).

Dzhafarov, E.N. (2007, March). A new geometry for subjective stimulus spaces. AFOSR Cognition and Decision Program Review (Fairborn , Ohio).

Dzhafarov, E.N. (2007, July). A new geometry of subjective stimulus spaces. Meeting of the Society for Mathematical Psychology (Irvine, California).

Kujala, J.V., & Dzhafarov, E.N. (2007, September). Tests for selectivity in the dependence of random variables on external factors. European Mathematical Psychology Group Meeting (Luxembourg).

Dzhafarov, E.N. (2007, November). Notions in topology, geometry, and probability growing from psychology. Purdue Winer Memorial Lectures (West Lafayette, Indiana).

Dzhafarov, E.N. (2008, July). The ancient sorites paradox and discrimination judgments. Meeting of the Society for Mathematical Psychology (Washington, DC).

Dzhafarov, E.N., & Kujala, J.V. (2008, July). Foundations of selective influence. Meeting of the Society for Mathematical Psychology (Washington, DC).

Kujala, J.V. , & Dzhafarov, E.N. (2008, July). Population-Level Tests of Selective Influence. Meeting of the Society for Mathematical Psychology (Washington, DC).

Dzhafarov, E.N. (2008, August). An ancient paradox for discrimination judgments. Meeting of the International Society for Psychophysics (Toronto, Canada).

Perry, L., & Dzhafarov, E.N. . (2009, August). Perceptual discrimination of two-dimensional stimuli: a test of matching regularity. Society for Mathematical Psychology Meeting, Amsterdam, The Netherlands.

Kujala, J.V. , & Dzhafarov, E.N. (2009, August). Reconciling Regular Minimality with Thurstonian-type Models. . Society for Mathematical Psychology Meeting, Amsterdam, The Netherlands.

Kiefer, T., Unlu, A., & Dzhafarov, E.N. (2009, August). Fechnerian Scaling in R. Society for Mathematical Psychology Meeting, Amsterdam, The Netherlands.

3.4 Software Developed within the Framework of the Grant

FSCAMDS (Fechnerian Scaling followed by Clustering and Multidimensional Scaling), a Matlab-based program available on http://www1.psych.purdue.edu/~ehtibar/Links.html. Developed by E.N. Dzhafarov and W. Rujikietgumjorn, in collaboration with H. Colonius.

R-language package "fechner" available on CRAN, http://www.jstatsoft.org/v31/i06/. Developed by A. Ünlü and T. Kiefer, in collaboration with E.N. Dzhafarov.

3.5 Keynote Addresses and Invited Conference Presentations Attributed to the Grant

Dzhafarov, E.N. (2008, February). Plenary lecture at the International Conference on Riemann-Finsler Geometry (Indianapolis, IN).

Dzhafarov, E.N. (2008, June). Foundations of Fechnerian Scaling. Training Course for senior researchers on "Theory and Methods of Measurement with Persons" (Genoa, Italy).

Dzhafarov, E.N. (2008, September). Plenary lecture at the European Mathematical Psychology Group Meeting (Graz, Austria).

3.6 Invited Talks Attributed to the Grant

Department of Psychology, Moscow State University (Lomonosov), Moscow, Russia (2007, September).

Independent University of Moscow, Moscow, Russia (2007, September).

Institute of Mathematics at University of Augsburg, Germany (2008, June).

Hope University, Liverpool, England (2009, May).

Swedish Collegium for Adavanced Studies, Uppsala, Sweden (2009, October).

Department of Philosophy, Uppsala University, Sweden (2009, November).

Department of Statistics, University of Dortmund, Germany (2009, December).

3.7 Symposia and Conferences Organized

Mathematical Theories of Perceptual Discrimination (2006, September). Meeting of the European Mathematical Psychology Group, in Brest, France (with Hans Colonius).

Mathematical Psychology as Applied Mathematics (2007, November). Purdue Winer Memorial Lectures, at Purdue University (with Oh-Sang Kwon).

Selective Influence (2008, July). Meeting of the Society for Mathematical Psychology (Washington, DC).

Perceptual Discrimination (2008, July). Meeting of the International Society for Psychophysics (Toronto, Canada).